Amphitheater School District End Of Year Algebra II Performance Assessment Review

This packet is intended to support student preparation and review for the Algebra II course concepts for the district common end of year assessment. This end of year assessment is aligned with the Arizona College and Career Readiness Standards. This is intended to be a guide only, it does not necessarily represent exact exam questions.

Review Problems for Final Exam

Determine whether the two expressions are equivalent.

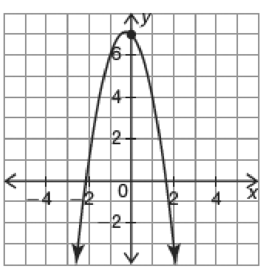
1. $(n^2+4n)-n^2$ and $4n$

2. $(5x+3)+3x^2-2$ and $(5x+1)+3x^2$

3. $(2x+1)^2-2x(x-3)$ and $6x^2+6x+2-(2x-1)^2$

Circle the function that matches each graph. Explain your reasoning.

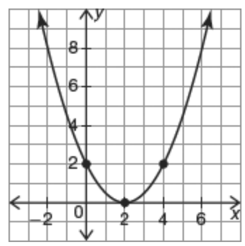
4.



$f(x) = 2x^2 - x + 7$ $\quad$ $f(x) = -x^2 - 2x + 7$

$f(x) = -2x^2 - x + 7$ $\quad$ $f(x) = -2x^2 - x - 2$

5.



$f(x) = -\frac{1}{2}(x-2)^2$ $\quad$ $f(x) = \frac{1}{2}(x-2)^2$

$f(x) = \frac{1}{2}(x-2)^2 + 2$ $\quad$ $f(x) = \frac{1}{2}(x+2)^2$

Each given function is in transformational function form $g(x) = Af(B(x-C)) + D$, where $f(x) = x^2$. Identify the values of *C* and *D* for the given function. Then, describe how the vertex of the given function compares to the vertex of *f*(*x*).

6. $g(x) = f(x-5) - 11$

7. $g(x) = f(x+2) + 3$

8. Graph the vertical dilation of $f(x) = x^2$ and tell whether the transformation is a vertical stretch or a vertical compression and if the graph includes a reflection.

$g(x) = -\frac{1}{2}x^2 - 3$

For each complex number, write its conjugate.

9. $3+5i$

10. $-7i$

11. Determine the product. $(4i-5)(4i+5)$

Use the discriminant to determine whether each function has real or imaginary zeros.

12. $f(x)=-3x^2+x-9$

13. $f(x)=9x^2-12x+4$

Use the vertex form of a quadratic equation to determine whether the zeros of each function are real or imaginary. Explain how you know.

14. $f(x)=-2(x-1)^2-5$

15. $f(x)=\frac{3}{4}(x+4)^2-6$

Factor each function over the set of real or imaginary numbers. Then, identify the type of zeros.

16. $n(x)=x^2-5x-14$

17. $g(x)=x^2+6x+10$

Use a coordinate plane to sketch a graph with the given characteristics. If the graph is not possible to sketch, explain why.

18. Characteristics:

- even degree
- increases to $x=-2$, then decreases to $x=0$, then increases to $x=2$, then decreases
- relative minimum at $y=1$
- two absolute maximums at $y=4$

19. Characteristics:

- as $x\to\infty, f(x)\to\infty$
  as $x\to-\infty, f(x)\to-\infty$
- *y*-intercept at $y=-2$
- three *x*-intercepts
- two relative extrema

Determine each function value using the Remainder Theorem. Explain your reasoning.

20. Determine $p(-2)$ if $p(x)=x^4-10x^3+8x^2+106x-105$.

21. Determine $p(-3)$ if $p(x)=2x^4+5x^3+8x^2+15x+6$.

Use the Factor Theorem to determine whether the given expression is a factor of each polynomial. Explain your reasoning.

22. Is $x-3$ a factor of $f(x)=4x^4-x^3-52x^2-35x+12$?

23. Is $3x+4$ a factor of $f(x)=3x^3+13x^2+18x+8$?

Factor completely.

24. $x^2-16x-36$

25. x^3+x^2-4x-4

26. x^4-29x^2+100

27. x^3-8y^3

28. $49x^2-4y^2$

29. $9x^4+42x^2y+49y^2$

30. $25x^2-35x+12$

Use the Rational Root Theorem to determine the possible rational roots.

31. $x^3+3x^2-18x-40$

32. $x^2-8x+17$

Perform each calculation. Describe any restriction(s) for the value of *x* and simplify the answer when possible.

33. $\frac{3}{x}+\frac{1}{x+1}$

34. $\frac{1}{x+3}-\frac{1}{x-3}$

35. $\frac{x+1}{x^2-16}-\frac{x}{x^2+7x+12}$

36. $\frac{1}{2x^2+3x-2}\cdot\frac{x^2-2x-8}{x-4}$

37. $\frac{x^2+6x+8}{3x+2}\div\frac{-x-4}{3x^2-x-2}$

Solve each rational equation. Describe any restrictions for the value of *x*. Check your answer(s) and identify any extraneous roots should they occur.

38. $\frac{9}{x-3}=\frac{27}{x^2-3x}+\frac{6}{x}$

39. $\dfrac{x-3}{x^2} = \dfrac{x-3}{x^2-1}$

Determine the inverse of each function.

40. $f(x) = 4x^2 - 1$

41. $f(x) = 3x^3 + 2$

42. $f(x) = (x+2)^4 - 16$

Perform the indicated operation. Be sure to simplify.

43. $\sqrt{16x^{10}y^8z^2}$

44. $5\sqrt{x}\,(\sqrt{x} + 3\sqrt{x}) - 10x$

45. $(5\sqrt{x})^2(4\sqrt{4x})$

46. $\dfrac{5\sqrt{2x^2y^2}}{3\sqrt[4]{32x^2y^5}}$

Solve each radical equation. Check for extraneous solutions.

47. $2x - 2 = \sqrt{x+2}$

48. $\sqrt[5]{3x-3} = 2$

49. $\sqrt{9x+3} - 11 = -8$

Simplify each expression completely, using only positive exponents.

50. $\dfrac{(2xy^4)^3}{(x^2y^3)^4}$

51. $(2x^{-2}y^3)^{-2}$

52. $\dfrac{x^3 \cdot y^{-5}}{x^{-4} \cdot y^2}$

Evaluate each log.

53. $\log_3 81$

54. $\log_4 4^5$

Write each logarithmic expression in expanded form.

55. $\log_5 10x^9y^7$

56. $\log_3 \dfrac{2}{x^2y^{10}}$

Write each logarithmic expression using a single logarithm. Evaluate the logarithm if possible.

57. $\log_3 6 + 5\log_3 x$

58. $\log_7 122 + 5\log_7 x + 8\log_7 y$

Convert each number of radians to degrees.

59. $\dfrac{7\pi}{4}$

60. $\dfrac{11\pi}{6}$

Convert each number of degrees to radians.

61. 240°

62. 135°

Evaluate each trigonometric function.

63. $\sin\dfrac{5\pi}{4}$

64. $\cos\dfrac{2\pi}{3}$

Calculate the tangent of each angle given the cosine and sine of the angle.

65. $\sin\theta = \dfrac{8}{17}, \cos\theta = \dfrac{15}{17}$

66. $\sin\theta = \dfrac{2\sqrt{5}}{5}, \cos\theta = \dfrac{\sqrt{5}}{5}$

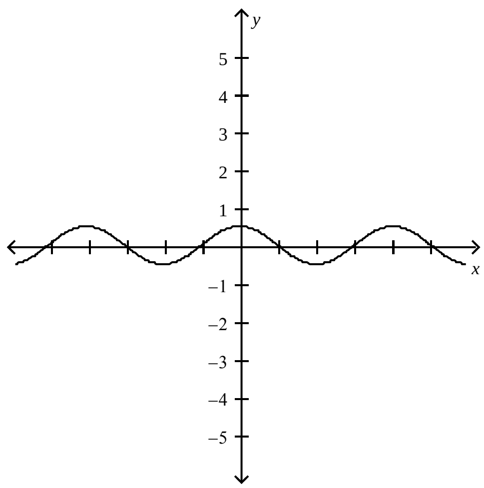
Determine each of the following in the first revolution. Determine *x* in radians.

67. $\cos x = \dfrac{\sqrt{3}}{2}$

68. $\tan x = 0$

69. Find the amplitude and period of $y = 3\sin x$

70. Find the amplitude and period of $y = \frac{1}{2}\cos 2x$ (each tick mark represents $\frac{\pi}{4}$



71. Write an arithmetic sequence that satisfies the criteria: The first term is 0.64. The common difference is -0.25. Write the first six terms of the sequence.

Determine whether each geometric series is convergent or divergent and calculate the series.

72. $\sum_{i=1}^{\infty} \left(\frac{2}{7}\right)^{i}$

73. $\sum_{i=0}^{\infty} 100(0.1)^{i}$

Solve each logarithmic equation. Check your answer(s).

74. $\log_{15}(x^2 - 2x) = 1$

75. $2\log_3 x - \log_3 8 = \log_3(x-2)$

76. $\log_4(x+3) + \log_4 x = 1$

77. $27 = 9^{2x-1}$

78. $2^{x+3} = 4^{x-2}$

79. $f(x) = \frac{1}{2}x + 3,\ g(x) = 4x + 10$ find $f(g(x))$

80. $f(x) = x^2 - 5,\ g(x) = 3x + 2$ find $f(g(x))$

81. $f(x) = 2x + 1,\ g(x) = x^3 - 3$ find $f(g(2))$

82. $f(x) = 3x^2 - 7,\ g(x) = 5x + 13$ find $f(g(-3))$

83. Find the 6th term in a geometric sequence given r=3and a first term of $\frac{1}{27}$

Review Problems for Final Exam
Answer Section

1. The expressions are equivalent.

Combine like terms. $(n^2+4n)-n^2=4n$.

2. $(5x+3)+3x^2-2$ $\qquad$ $(5x+1)+3x^2$

$3x^2+5x+1$ $\qquad$ $3x^2+5x+1$

The expressions are equivalent.

3. $(2x+1)^2-2x(x-3)$ $\qquad$ $6x^2+6x+2-(2x-1)^2$

$4x^2+4x+1-2x^2+6x$ $\qquad$ $6x^2+6x+2-(4x^2-4x+1)$

$2x^2+10x+1$ $\qquad$ $6x^2+6x+2-4x^2+4x-1$

$2x^2+10x+1$

The expressions are equivalent.

4. $f(x)=-2x^2-x+7$

The *a*-value is negative so the parabola opens down. Also, the *y*-intercept is (0, 7).

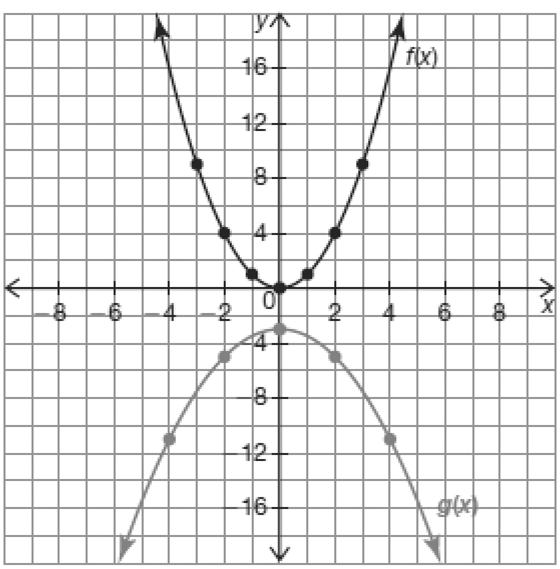
5. $f(x)=\frac{1}{2}(x-2)^2$

The *a*-value is positive so the parabola opens up. The minimum point is (2, 0).

6. The *C*–value is 5 and the *D*–value is -11, so the vertex will be shifted 5 units to the right and 11 units down to $(5,-11)$.

7. The *C*–value is -2 and the *D*–value is 3 so the vertex will be shifted 2 units to the left and 3 units up to $(-2,3)$.

8.



vertical compression and reflection over the x-axis, and shifted down 3.

9. $3-5i$

10. $7i$

11. $(4i-5)(4i+5)=16i^2+20i-20i-25$

$$=16(-1)-25$$

$$=-41$$

12. $b^2-4ac=1^2-4(-3)(-9)$

$$=1-108$$

$$=-107$$

The discriminant is negative, so the function has imaginary zeros.

13. $b^2-4ac=(-12)^2-4(9)(4)$

$$=144-144$$

$$=0$$

The discriminant is zero, so the function has real zeros (double roots).

14. Because the vertex $(-1,-5)$ is below the *x*-axis and the parabola is concave down $(a<0)$, it does not intersect the *x*-axis. So, the zeros are imaginary.

15. Because the vertex $(-4,-6)$ is below the *x*-axis and the parabola is concave up, it intersects the *x*-axis. So, the zeros are real.

16. $n(x)=(x-7)(x+2)$

$x=7, x=-2$

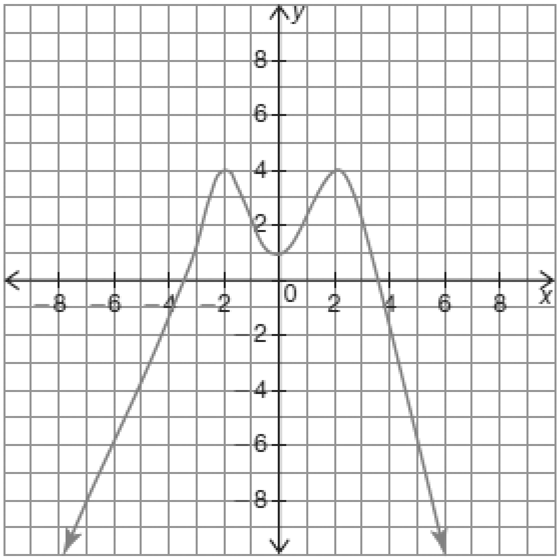
The function *n*(*x*) has two real zeros.

17. $g(x)=\left[x-(-3-i)\right]\left[x-(-3+i)\right]$

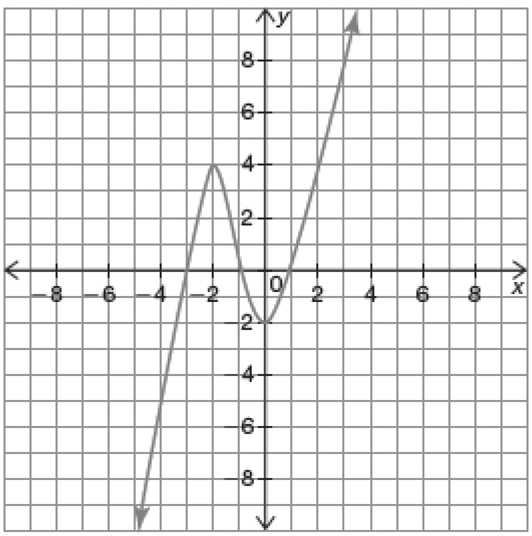
$x=-3-i, x=-3+i$

The function *g*(*x*) has two imaginary zeros.

18.



19.



20.

−2	1	−10	8	106	−105
		−2	24	−64	−84
	1	−12	32	42	−189

When *p*(*x*) is divided by $x+2$, the remainder is -189. So, by the Remainder Theorem $p(-2)=-189$.

21.

−3	2	5	8	15	6
		−6	3	−33	54
	2	−1	11	−18	60

When *p*(*x*) is divided by $x+3$, the remainder is 60. So, by the Remainder Theorem $p(-3)=60$.

22. If $x-3$ is a factor of *f*(*x*), then by the Factor Theorem $f(3)=0$.

$$f(3)=4(3)^4-(3)^3-52(3)^2-35(3)+12$$
$$f(3)=324-27-468-105+12$$
$$f(3)=-264$$

When *f*(*x*) is evaluated at 3, the result is -264. According to the Factor Theorem $x-3$ is not a factor of *f*(*x*).

23. If $3x+4$ is a factor of *f*(*x*), then by the Factor Theorem $f\left(-\frac{4}{3}\right)=0$.

$$f\left(-\frac{4}{3}\right)=3\left(-\frac{4}{3}\right)^3+13\left(-\frac{4}{3}\right)^2+18\left(-\frac{4}{3}\right)+8$$
$$f\left(-\frac{4}{3}\right)=-\frac{64}{9}+\frac{208}{9}-24+8$$
$$f\left(-\frac{4}{3}\right)=0$$

When *f*(*x*) is evaluated at $-\frac{4}{3}$, the result is 0. According to the Factor Theorem $3x+4$ is a factor of *f*(*x*).

24. $x^2-16x-36=(x+2)(x-18)$

25. $x^3+x^2-4x-4=x^2(x+1)-4(x+1)$

$$=(x^2-4)(x+1)$$
$$=(x+2)(x-2)(x+1)$$

26. $x^4-29x^2+100=(x^2-25)(x^2-4)$

$$=(x-5)(x+5)(x-2)(x+2)$$

27. $a^3-b^3=(a-b)(a^2+ab+b^2)$

$$x^3-8y^3=(x)^3-(2y)^3$$
$$=(x-2y)(x^2+2xy+4y^2)$$

28. $a^2-b^2=(a+b)(a-b)$.

$$49x^2-4y^2=(7x)^2-(2y)^2$$
$$=(7x-2y)(7x+2y)$$

29. $a^2+2ab+b^2=(a+b)^2$

$$9x^4+42x^2y+49y^2=(3x^2)^2+2(3x^2)(7y)+(7y)^2$$
$$=(3x^2+7y)^2$$

30. $25x^2-35x+12=(5x)^2-7(5x)+12$

Let $z=5x$

$$=z^2-7z+12$$
$$=(z-3)(z-4)$$
$$=(5x-3)(5x-4)$$

31. • Possible rational roots:

$p=\pm1, \pm2, \pm4, \pm5, \pm8, \pm10, \pm20, \pm40$

$q=\pm1$

$\dfrac{p}{q}=\pm1, \pm2, \pm4, \pm5, \pm8, \pm10, \pm20, \pm40$

32. • Possible rational roots:

$p=\pm1, \pm17$

$q=\pm1$

$\dfrac{p}{q}=\pm1, \pm17$

33. $\dfrac{3}{x}+\dfrac{1}{x+1}=\dfrac{3(x+1)}{x(x+1)}+\dfrac{1(x)}{(x+1)(x)}$

$$=\frac{3x+3}{x(x+1)}+\frac{x}{x(x+1)}$$
$$=\frac{4x+3}{x(x+1)};\ x\neq-1, 0$$

34. $\dfrac{1}{x+3}-\dfrac{1}{x-3}=\dfrac{1(x-3)}{(x+3)(x-3)}-\dfrac{1(x+3)}{(x-3)(x+3)}$

$$=\frac{x-3}{(x+3)(x-3)}-\frac{x+3}{(x+3)(x-3)}$$
$$=\frac{-6}{(x+3)(x-3)};\ x\neq\pm3$$

35. $\dfrac{x+1}{x^2-16}-\dfrac{x}{x^2+7x+12}=\dfrac{x+1}{(x-4)(x+4)}-\dfrac{x}{(x+3)(x+4)}$

$$=\frac{(x+1)(x+3)}{(x-4)(x+4)(x+3)}-\frac{x(x-4)}{(x+3)(x+4)(x-4)}$$
$$=\frac{x^2+4x+3}{(x-4)(x+4)(x+3)}-\frac{x^2-4x}{(x+3)(x+4)(x-4)}$$
$$=\frac{8x+3}{(x-4)(x+4)(x+3)};\ x\neq-4,-3,4$$

36. Restrictions: $x \neq -2, \frac{1}{2}, 4$

$$\begin{aligned}\frac{1}{2x^2+3x-2}\cdot\frac{x^2-2x-8}{x-4} &= \frac{1}{(x+2)(2x-1)}\cdot\frac{(x+2)(x-4)}{x-4}\\ &= \frac{1}{\cancel{(x+2)}(2x-1)}\cdot\frac{\cancel{(x+2)}\cancel{(x-4)}}{\cancel{x-4}}\\ &= \frac{1}{2x-1}\end{aligned}$$

37. Restrictions: $x \neq -4, -\frac{2}{3}, 1$

$$\begin{aligned}\frac{x^2+6x+8}{3x+2}\div\frac{-x-4}{3x^2-x-2} &= \frac{x^2+6x+8}{3x+2}\cdot\frac{3x^2-x-2}{-x-4}\\ &= \frac{(x+4)(x+2)}{3x+2}\cdot\frac{(3x+2)(x-1)}{-1(x+4)}\\ &= \frac{\cancel{(x+4)}(x+2)}{\cancel{3x+2}}\cdot\frac{\cancel{(3x+2)}(x-1)}{-1\cancel{(x+4)}}\\ &= \frac{x^2+x-2}{-1}\\ &= -x^2-x+2\end{aligned}$$

38. Restrictions: $x \neq 0, 3$

$$\begin{aligned}\frac{9}{x-3} &= \frac{27}{x(x-3)}+\frac{6}{x}\\ x(x-3)\left[\frac{9}{x-3}\right] &= x(x-3)\left[\frac{27}{x(x-3)}+\frac{6}{x}\right]\\ 9x &= 27+6x-18\\ 3x &= 9\\ x &= 3\end{aligned}$$

However, $x \neq 3$ because it is a restriction on the variable and thus is an extraneous root. This equation has no solution.

39. Check $x = 3$.

Restrictions: $x \neq -1, 0, 1$

$$(x-3)(x^2-1) = x^2(x-3)$$
$$x^3 - 3x^2 - x + 3 = x^3 - 3x^2$$
$$-x + 3 = 0$$
$$-x = -3$$
$$x = 3$$

$$\frac{3-3}{(3)^2} \stackrel{?}{=} \frac{3-3}{(3)^2 - 1}$$
$$\frac{0}{9} \stackrel{?}{=} \frac{0}{8}$$
$$0 = 0 \quad \checkmark$$

40. $f^{-1}(x) = \pm\frac{1}{2}\sqrt{x+1}$

41. $f^{-1}(x) = \sqrt[3]{\frac{x-2}{3}}$

42. $f^{-1}(x) = \pm\sqrt[4]{x+16} - 2$

43. $= (16x^{10}y^8z^2)^{\frac{1}{2}} = 4y^4\left|x^5 z\right|$

44. $= 5x + 15x - 10x = 10x$

45. $= 25x(8\sqrt{x}) = 200x\sqrt{x}$

46. $= \dfrac{5(2)^{\frac{1}{2}} y|x|}{3(2)^{\frac{5}{4}} x^{\frac{1}{2}} y^{\frac{5}{4}}} = \dfrac{5x^{\frac{2}{4}}}{3(2)^{\frac{3}{4}} y^{\frac{1}{4}}} = \dfrac{5}{3} \cdot \sqrt[4]{\dfrac{x^2}{8y}}$

y must be positive because of $\sqrt[4]{y^5}$.

47.

$$4x^2 - 8x + 4 = x + 2$$
$$4x^2 - 9x + 2 = 0$$
$$(4x-1)(x-2) = 0$$
$$x = \frac{1}{4},\ x = 2$$

Solution: $x = 2$

$$2\left(\frac{1}{4}\right) - 2 \stackrel{?}{=} \sqrt{\left(\frac{1}{4}\right) + 2}$$
$$\frac{-3}{2} \stackrel{?}{=} \sqrt{\frac{9}{4}}$$
$$\frac{-3}{2} \neq \frac{3}{2}$$

Extraneous Root

$$2(2) - 2 \stackrel{?}{=} \sqrt{(2) + 2}$$
$$2 \stackrel{?}{=} 4$$
$$2 = 2$$

48. $(\sqrt[5]{3x-3})^5 = (2)^5$

$$3x - 3 = 32$$
$$3x = 35$$
$$x = \frac{35}{3}$$

49. $\sqrt{9x+3}=3$

$(\sqrt{9x+3})^2=(3)^2$

$9x+3=9$

$9x=6$

$x=\frac{2}{3}$

50. $\frac{-8x^3y^{12}}{x^8y^{12}}=\frac{8}{x^5}$

51. $\left(\frac{2y^3}{x^2}\right)^{-2}=\left(\frac{x^2}{2y^3}\right)^2=\frac{x^4}{4y^6}$

52. $\frac{x^3x^4}{y^5y^2}=\frac{x^7}{y^7}$

53. 4

54. 5

55. $\log_5 10+9\log_5 x+7\log_5 y$

56. $\log_3 2-2\log_3 x-10\log_3 y$

57. $\log_3 6x^5$

58. $\log_7 122x^5y^8$

59. $\frac{7\pi}{4}\times\frac{360^o}{2\pi}=315^c$

60. $\frac{11\pi}{6}\times\frac{360^o}{2\pi}=330^c$

61. $240^o\times\frac{2\pi}{360^o}=\frac{4\pi}{3}$

62. $135^o\times\frac{2\pi}{360^o}=\frac{3\pi}{4}$

63. $\sin\frac{5\pi}{4}=-\frac{\sqrt{2}}{2}$

64. $\cos\frac{2\pi}{3}=-\frac{1}{2}$

65. $\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{8}{17}\div\frac{15}{17}=\frac{8}{17}\times\frac{17}{15}=\frac{8}{15}$

66. $\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{2\sqrt{5}}{5}\div\frac{\sqrt{5}}{5}=\frac{2\sqrt{5}}{5}\times\frac{5}{\sqrt{5}}=2$

67. $x=\frac{5\pi}{6}$ radians, $\frac{7\pi}{6}$ radians

68. $x=0$ radians, π radians, 2π radians

69. amplitude $=3$ period 2pi

70. amplitude $=\frac{1}{2}$ period = pi

71. 0.64, 0.39, 0.14, -0.11, -0.36, -0.61

72. $r=\frac{2}{7}$

The series is convergent because the common ratio is between 0 and 1.

73. $r=0.1$

The series is convergent because the common ratio is between 0 and 1.

74. $\log_{15}(x^2-2x)=1$

$$15^1=x^2-2x$$
$$0=x^2-2x-15$$
$$0=(x+3)(x-5)$$
$$x=-3,5$$

Check:

$$\log_{15}((-3)^2-2(-3))\stackrel{?}{=}1$$
$$\log_{15}(9+6)\stackrel{?}{=}1$$
$$\log_{15}15=1$$

$$\log_{15}(5^2-2(5))\stackrel{?}{=}1$$
$$\log_{15}(25-10)\stackrel{?}{=}1$$
$$\log_{15}15=1$$

75. $2\log_3 x-\log_3 8=\log_3(x-2)$

$$\log_3 x^2-\log_3 8=\log_3(x-2)$$
$$\log_3\left(\frac{x^2}{8}\right)=\log_3(x-2)$$
$$\frac{x^2}{8}=x-2$$
$$x^2=8x-16$$
$$x^2-8x+16=0$$
$$(x-4)^2=0$$
$$x=4$$

Check:

$$2\log_3 4 - \log_3 8 \stackrel{?}{=} \log_3(4-2)$$

$$\log_3 16 - \log_3 8 \stackrel{?}{=} \log_3 2$$

$$\log_3\left(\frac{16}{8}\right) \stackrel{?}{=} \log_3 2$$

$$\log_3 2 = \log_3 2$$

76. $\log_4(x+3) + \log_4 x = 1$

$$\log_4(x(x+3)) = 1$$

$$x(x+3) = 4^1$$

$$x^2 + 3x = 4$$

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$x = -4, 1$$

Check:

-4 is an extraneous solution.

$$\log_4(1+3) + \log_4 1 \stackrel{?}{=} 1$$

$$\log_4 4 + \log_4 1 \stackrel{?}{=} 1$$

$$1 + 0 \stackrel{?}{=} 1$$

$$1 = 1$$

77. x=5/4
78. x=7
79. 2x+8
80. $9x^2 + 12x - 1$
81. 11
82. 5
83. 9